



Privacy Preserving Analytics

PSI + Analytics using Homomorphic Encryption

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Topics

- Purpose
- Homomorphic Encryption (short introduction)
- PSI using HE
- Analytics using HE
- Analytics using HE + helper
- Q&A



Purpose



High level purpose

Private Set Intersection:

- Determine **set intersection** of datasets from **multiple owners** while **preserving input privacy**

e.g. 'How many people are customers of both company A and B without revealing specific customers to each other?'



Privacy Preserving Analytics:

- Perform **statistical analysis** on datasets from **multiple owners** while **preserving input privacy**

e.g. 'What's the average spending of customers of company A who are also customer of company B?'



Preserving input privacy

Several privacy preserving technologies:

- Trusted Execution Environment
- Garbled circuits
- Secret Sharing
- **Homomorphic Encryption**
- ...
- Combination(s) of above technologies



Homomorphic Encryption



Homomorphic Encryption

- **Computations on encrypted data possible without decrypting first**
- **Result after decrypting equals equivalent computation on unencrypted cleartext :**

$$\text{Decrypt}(\text{Function}(\text{Encrypt}(x))) = \text{Function}(x)$$

(actual function in the encrypted domain is not identical to function in unencrypted domain)

- **Asymmetric:** different keys for encrypting and decrypting

→ Enables 'outsourcing' of computations on your sensitive data to others



2 Types of HE

- **Partial**, only **single type** of operation possible, e.g.:
 - Multiplicative (**ciphertext** · **ciphertext**)
 - Additive (**ciphertext** + **ciphertext**)
- **Fully**, both **additive and multiplicative**
 - Severe performance drop
 - Very large ciphertexts and keys
 - Limited arithmetic circuit depth
 - Added complexity



Homomorphic Encryption: Important aspects

- Ciphertexts are:
 - large, random-looking numbers
 - **re-randomizable** (multiply by encrypted 1 or add encrypted 0...)
 - **indistinguishable!**

Plaintext: 3

Ciphertext: 1736734601920938409279237659872346123871002093878777742341

Plaintext: 3

Ciphertext: 9928374645102937462812384760092374987623466277478488222164



PSI using HE



Concept – key aspects

- **Set membership** of a private set can be expressed **numerically** (**1 = in my set, 0 = not in my set**)
- HE encrypted **1's and 0's** are **indistinguishable**
- HE encrypted **set membership** can be **added numerically** (counting, using 'simple' additive HE Scheme)
- Each party **replaces** set **entries** for entities **not in their set** by encrypted 0's
- **Summing** encrypted 1's and 0's **creates intersection count**



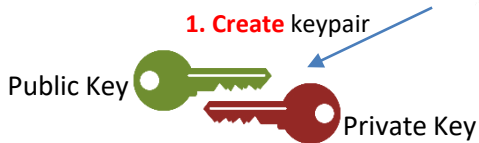
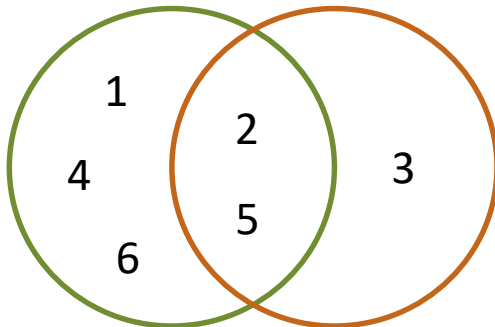
Example



P1



P2



ID	P1	P1P2
1	1	1
2	1	1
3	0	0
4	1	1
5	1	1
6	1	1

2. Create encrypted initial table

3. Send table + public key



Plaintext: 1 or 0

Ciphertext: 'Random' 136253748903876725241038746...

Encrypted columns!

4. Link records: matching ID's

5. Create intersection: zeroing cells for non-matching ID's

ID	P1	P1P2	P2
1	1	± 0	0
2	1	1	1
3	0	0	1
4	1	± 0	0
5	1	1	1
6	1	± 0	0
Σ	5	2	3

6. Sum columns

Encrypted columns!

	P1	P1P2	P2
Σ	5	2	3

7. Return encrypted counts (bottom row)

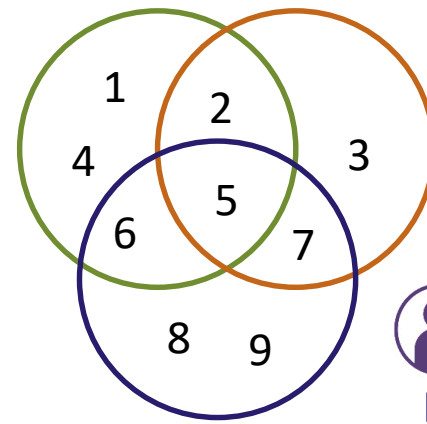
6. Decrypt and reveal counts



Extending concept: more parties



P1



P2



P3

ID	P1	P1P2	P1P3	P1P2P3
1	1	1	1	1
2	1	1	1	1
3	0	0	0	0
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0

ID	P1	P1P2	P1P3	P1P2P3	P2	P2P3
1	1	±0	1	±0	0	0
2	1	1	1	1	1	1
3	0	0	0	0	1	1
4	1	±0	1	±0	0	0
5	1	1	1	1	1	1
6	1	±0	1	±0	0	0
7	0	0	0	0	1	1
8	0	∅0	0	∅0	0	0
9	0	∅0	0	∅0	0	0

ID	P1	P1P2	P1P3	P1P2P3	P2	P2P3	P3
1	1	0	±0	∅0	0	∅0	0
2	1	1	±0	±0	1	±0	0
3	0	0	∅0	∅0	1	±0	0
4	1	0	±0	∅0	0	∅0	0
5	1	1	1	1	1	1	1
6	1	0	1	0	0	0	1
7	0	0	0	0	1	1	1
8	0	0	0	0	0	0	1
9	0	0	0	0	0	0	1
Σ	5	2	2	1	4	2	5

P1 encrypt → P2 replace → P3 replace and sum →
P1 decrypt aggregates + broadcast

PPA using HE



Concept – key aspects

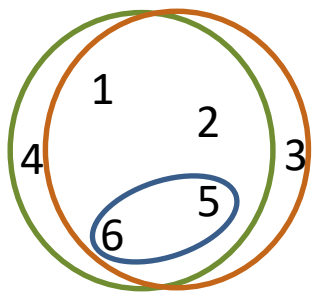
- **Builds on PSI example**
- **All numbers are indistinguishable** (not only 0 and 1)
- **Enables passing encrypted fact data to other parties**
- **Parties filter / select rows conditionally based on own facts / data**
- **Other parties can manipulate facts ‘blinded’ under HE** (e.g. replacing by a specific number or adding/multiplying etc.)
- **Last party aggregates under HE**



Example



P1



P2

Calculate **average income** for people with mobile roaming costs > 200

ID	P1: Income
1	1700
2	2300
3	0
4	1500
5	5200
6	6100

- Can also be **extended** to parties > 2
- **Any party** can act as **filter** or **aggregatable** party
- **Complex analytics** require **FHE** scheme and/or **multiple communication rounds**

ID	P1: Income	P2: Mobile Roaming Costs	P2: Filter (count)
1	1700	50	0
2	2300	40	0
3	0	160	0
4	1500	0	0
5	5200	300	1
6	6100	250	1
Σ	11300		2



	Income sum	People count
Σ	11300	2



$$\text{Avg} = \text{Decrypt}(11300) / 2$$

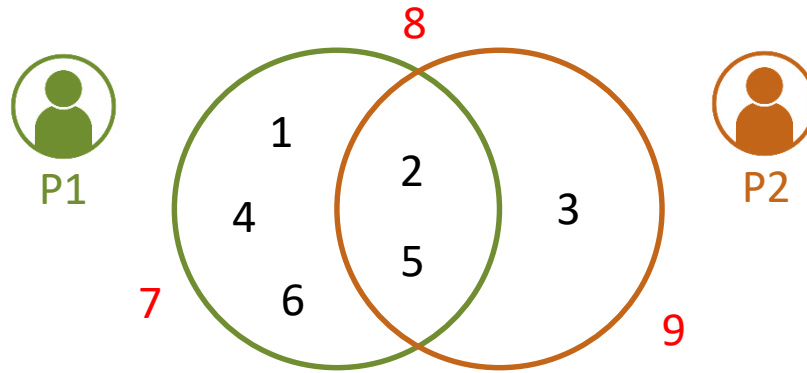
↑
Encrypted column!



PPA using HE + helper party



Limitations



- Some population disclosure inevitable...
- Initial population should not be sensitive
 - Union of P1 and P2 (if both not sensitive)
 - P1 or P2 (if only P2 or P1 is sensitive)
 - Superset of P1 and P2 (if P1 and P2 sensitive: e.g. 'all people in country')
- But: larger population → lower performance (ciphertext expansion & data exchange, more computations etc.)
- What if P1 and P2 sensitive and superset not viable?? → **Helper party**



Concept – key aspects

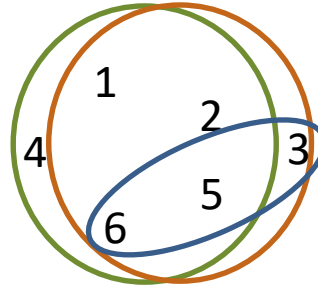
- **Data parties**
 - **Jointly create shared keypair**
 - **Filter and encrypt** own data locally
 - **Pseudonymize ID's**
- **Helper party**
 - performs **intersection + aggregate calculations**
 - **Sends encrypted aggregates back** to data parties **for decryption**
- **No party** learns **other population, only sizes**
- **Data parties** should **not collude** with **helper party**



Example

Calculate **average income** for people with mobile roaming costs > 200

ID	P1: Income	PID	P1: Income
1	1700	@a3	1700
2	2300	5%u	2300
4	1500	22>	1500
5	5200	!ab	5200
6	6100	?o9	6100



ID	PID	P2: Mobile Roaming Costs
1	@a3	50
2	5%u	40
3	gd2	210
5	!ab	300
6	?o9	250

PID	P2: Mobile Roaming Costs
gd2	210
!ab	300
?o9	250

	P1: income	Matches
Σ	11300	2



Avg: 11300 / 2

Send to P1 & P2



Helper

Match ID's and aggregate

PID	P1: Income	P2: Mobile Roaming Costs
@a3	1700	-
gd2	-	210
5%u	2300	-
22>	1500	-
!ab	5200	300
?o9	6100	250
Σ	11300	

Thank you!

Questions?

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Facts that matter