I. M-estimators for regression (with two weight functions and scale parameters)

II. M-estimators for generalised ratio model (with Tukey’s biweight function and average absolute deviation (AAD) for scale parameter)

III. Modified Stahel-Donoho (MSD) estimator

### Features
- RLS (Iteratively Reweighted Least Squares) based on Bienias et al. (1997) in Statistical Data Editing 2 – Methods and Techniques by Tukey’s biweight function with AAD scale.
- R functions by Huber’s weight function, and also with MAD scale are implemented for comparison
- Tukey’s biweight function is more suitable with AAD scale.

### Algorithm
- Prepare random orthogonal bases for projection
- Estimate the robust mean vector and the covariance matrix by weighting
- Trim weights of observations at the edge to control their influence on estimation
- Projection on to principal components to improve initial weights
- Weighted Principal components analysis

### Deference between the regression model and the ratio model

**Regression model**

\[ y_i = \alpha + \beta x_i + \varepsilon_i \]

**Ratio model**

\[ y_i = \frac{\hat{y}_i}{x_i} + \varepsilon_i \]

### Robustification and generalisation

1. Making the error term homoscedastic
   \[ \varepsilon \sim N(0, \sigma^2) \]

2. Robustification
   \[ \hat{\beta}_{rob} = \frac{\sum w_i y_i x_i}{\sum w_i x_i^2} \]

- Quasi-residual: \[ \varepsilon_i = \frac{y_i x_i}{\sqrt{\sum w_i x_i^2}} - \hat{\beta}_{rob} \sqrt{\sum w_i x_i^2} \]
- Weight function: Tukey’s biweight (c=8)
- Scale parameter: \[ \delta_{AAD} = \frac{1}{n} \sum |r_i| \]

- AAD: average absolute deviation
- MAD: median absolute deviation

### File names available at https://github.com/kazwd2008/IRLS

<table>
<thead>
<tr>
<th>File name</th>
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### Practical application

- 2016 Economic Census for Business Activities
  - Robust estimator for generalised ratio model (y=1/2)
- 2019 Unincorporated Enterprise Survey
  - Robust estimator for generalised ratio model (y=1/2)
  - MSD estimators (single core version)

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**References**

- WADA Kazumi (Tsuda University) and TSUBAKI Hiroe (The Institute of Statistical Mathematics (ISM))