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Sampling coordination of business surveys conducted by INSEE

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Random numbers

Very often national statistics agencies use the permanent random numbers technique for the sampling coordination of business surveys.

Each unit k of the population (included the new units) is independently assigned a number ω_k, selected according to the uniform distribution in the interval [0,1 [.

Poisson sampling : selection of the units k whose numbers belong to the interval [d, d + π_k[, π_k = probability of inclusion of the unit k

Simple random sampling (SRS) : selection of the n units with the lowest numbers ω_k superior to d.

Constant shift method : J panels (j = 1 …J)

Starting point for panel j at the date a : $d_{j,a} = d_{j,1} + (a-1)c, \quad a \geq 1$

Definition of a coordination function

Coordination function g = measurable application from [0,1[to [0,1[which preserves uniform probability :

if P is the uniform probability on [0,1[, then the image probability

$$P^g = P.$$

$\rightarrow$ for any interval I = [a,b [included in [0,1[:

$$P(g^{-1}(I)) = P^g(I) = P(I) = b - a$$

Selection of the units

For each unit k in the sampling frame :

- a permanent random number ω_k,
- a coordination function $g_{k,t}$ that changes at each sampling t = 1, 2, …

1. Poisson sampling

Selection of the units k such that $g_k(\omega_k) \in [0, \pi_k[$
where π_k = probability of inclusion of the unit k

$$P(k \in S) = P(g_k(\omega_k) \in [0, \pi_k[) = P^{g_k}([0, \pi_k[) = P([0, \pi_k[) = \pi_k$$

and the drawings are independent.

Selection of the units

2. SSRS

Within a stratum, selection of the n units k associated with the n smallest numbers $g_k(\omega_k)$.

Since

- P^{g_k} = uniform probability P sur [0,1[for each k

- and the n numbers (ω_k) are drawn independently from P

→ the n numbers $g_k(\omega_k)$ are drawn independently from P

→ the n smallest numbers $g_k(\omega_k)$ give a simple random sample of size n in the stratum.

Example : the constant shift method

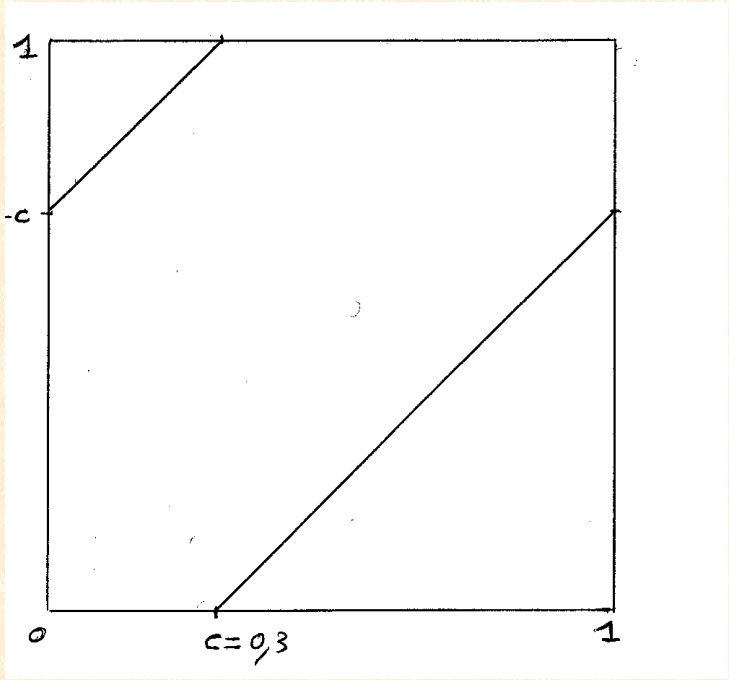
Let $d_1 = 0$ and $d_2 = c$. We define the coordination functions :

$$g_{k,1}(\omega_k) = \omega_k \qquad g_{k,2}(\omega_k) = \omega_k - c \pmod 1$$

Then :

$$k \in S_1 \Leftrightarrow \omega_k \in [0, \pi_{k,1}[\Leftrightarrow g_{k,1}(\omega_k) \in [0, \pi_{k,1}[$$

$$k \in S_2 \Leftrightarrow \omega_k \in [c, c + \pi_{k,2}[\Leftrightarrow g_{k,2}(\omega_k) \in [0, \pi_{k,2}[$$



A step by step procedure reflecting response burdens

$\boldsymbol{\omega} = (\ldots\ \omega_k \ldots)$ = vector of random numbers given to the population units.

$I_{k,t}(\boldsymbol{\omega})$ = indicator function, equal to 1 if the values in $\boldsymbol{\omega}$ lead to select the unit k in the sampling t, and 0 otherwise :

$$k \in S_t \Leftrightarrow I_{k,t}(\boldsymbol{\omega}) = 1$$

$\gamma_{k,t}$ = response burden of a questioned enterprise k at survey t

Effective burden = random variable $\gamma_{k,t}(\boldsymbol{\omega}) = \gamma_{k,t}\ I_{k,t}(\boldsymbol{\omega})$

Cumulative burden for unit k : $\Gamma_{k,t}(\boldsymbol{\omega}) = \sum_{u \le t} \gamma_{k,u} . I_{k,u}(\boldsymbol{\omega})$

<u>Principle</u> : to define the coordination functions $g_{k,t}$ for the selection of sample S_t using $\Gamma_{k,t-1}$:

$$\Gamma_{k,t-1}(\boldsymbol{\omega_1}) < \Gamma_{k,t-1}(\boldsymbol{\omega_2}) \Rightarrow g_{k,t}(\omega_{k,1}) < g_{k,t}(\omega_{k,2})$$

Two difficulties

1. to substitute for $\Gamma_{k,t}(\boldsymbol{\omega})$ a function of ω_k only, denoted $\Gamma'_{k,t}(\omega_k)$, that closely approximates $\Gamma_{k,t}(\boldsymbol{\omega})$.

Poisson sampling : $I_{k,t}(\boldsymbol{\omega})$ depends only on ω_k (indicator function of an interval of length π_k), and can be denoted $I_{k,t}(\omega_k)$
$\rightarrow$ $\Gamma_{k,t}(\boldsymbol{\omega})$ depends only on ω_k , and can be denoted $\Gamma'_{k,t}(\omega_k)$.

SSRS : $I_{k,t}(\boldsymbol{\omega})$ depends on all coordinates of the vector $\boldsymbol{\omega}$, but "primarily" on coordinate ω_k : if we select the $\boldsymbol{n}$ units among $\boldsymbol{N}$ with the $\boldsymbol{n}$ smallest values ω_j, it will be equal to 1 for values of ω_k near to 0, regardless of the values of the other coordinates $\rightarrow$ we will be able to replace $I_{k,t}(\boldsymbol{\omega})$ with an approximation $I'_{k,t}(\omega_k)$, and therefore to replace $\Gamma_{k,t}(\boldsymbol{\omega})$ with an approximation $\Gamma'_{k,t}(\omega_k)$.

2. to define the coordination function $g_{k,t}$ such that :

$$\Gamma'_{k,t-1}(\omega_{k,1}) < \Gamma'_{k,t-1}(\omega_{k,2}) \Rightarrow g_{k,t}(\omega_{k,1}) < g_{k,t}(\omega_{k,2})$$

Construction of a coordination function

$C_{k,t}(\omega_k)$ = criterion such that the smaller is the criterion, the larger is the probability of selection for unit k at sampling t.

We drop the subscripts k and t. C is supposed to be a bounded measurable function :

$$\omega \in [0,1[\rightarrow C(\omega) \in \mathbb{R}$$

Idea : to associate to this criterion a coordination function g_C such that :

$$C(\omega_1) < C(\omega_2) \Rightarrow g_C(\omega_1) < g_C(\omega_2) \quad (1)$$

P^C = image probability of P under C, F_C the distribution function of C.

The coordination function is built from $G_C = F_C(C)$:

$$G_C(\omega) = P^C(]-\infty, C(\omega)[) = P(C^{-1}]-\infty, C(\omega)[) = P(u | C(u) < C(\omega))$$

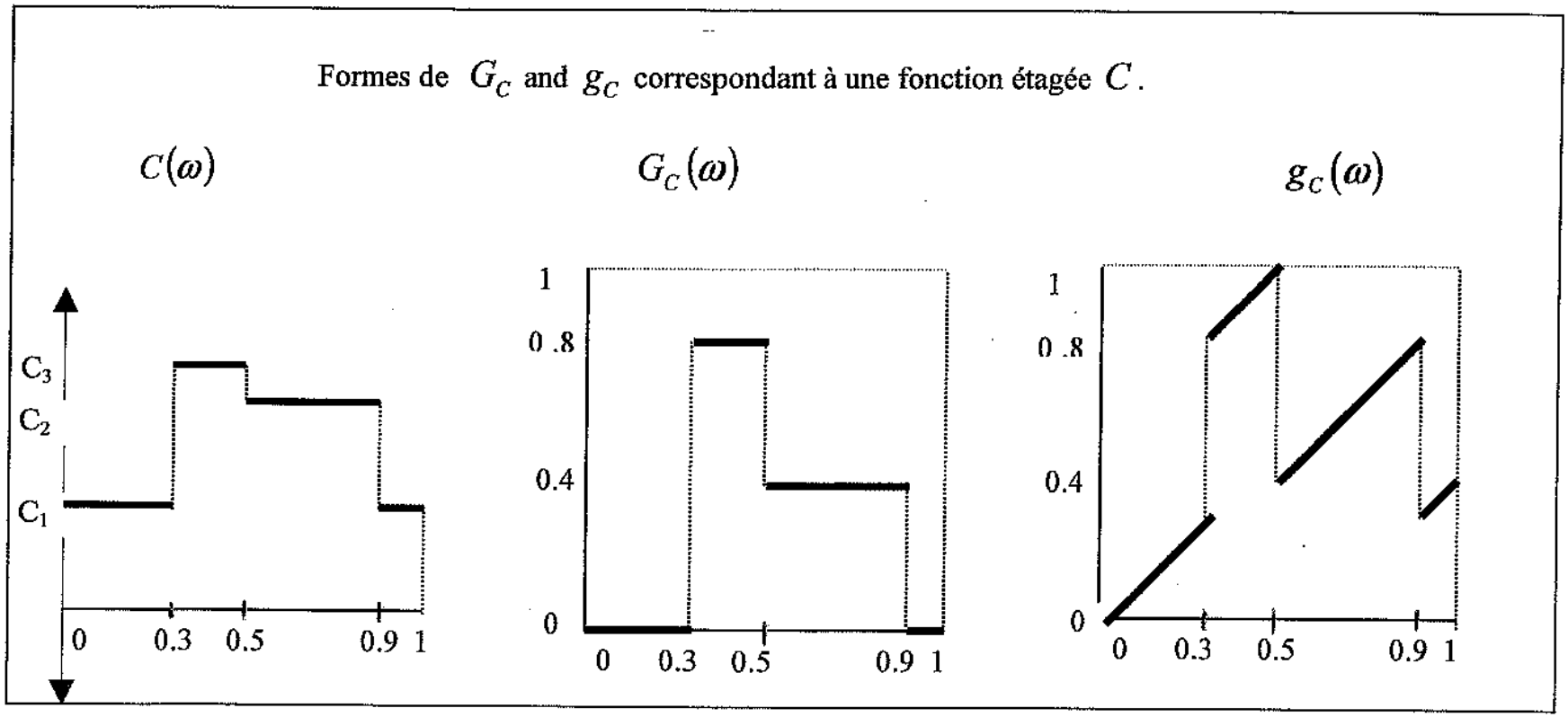
The way to derive g_C from G_C depends on whether or not C has *levels*.

Definition : we call a *level* of criterion C any inverse image of a real number y such that $P^C(y) = P(A) > 0$
i.e. C has *levels* when horizontal line segments form part of the graph of C.

Properties of G_C

- The range of G_C is included in [0,1[
- G_C has the same *levels* as C
- G_C verifies implication (1)
- For every y in the range of G_C, we have $P\left(u \middle| G_C(u) < y\right) = y$
- If C has no *level*, G_C is a coordination function.

If C has at least one level, the range of function G_C is strictly included in [0,1[. We have to deduce from G_C another function, denoted g_C, such that the range of g_C is equal to [0,1[.



Coordination function with several criteria

If criterion C has levels A_i, we can introduce secondary criteria C_i corresponding to each level.

The coordination function verifies the following conditions:

$$\forall\ \omega_1, \omega_2 \in A_i \quad C_i(\omega_1) < C_i(\omega_2) \Rightarrow g(\omega_1) < g(\omega_2)$$

Application to Poisson sampling

Initialization : $\Gamma_{k,0}(\omega_k) = 0 \quad g_{k,1}(\omega_k) = \omega_k$

$$I_{k,1}(\omega_k) = \mathbb{1}_{[0,\pi_{k,1}[}(\omega_k) \quad \Gamma_{k,1}(\omega_k) = \gamma_{k,1}\, \mathbb{1}_{[0,\pi_{k,1}[}(\omega_k)$$

For sample S_t, we choose a coordination function $g_{k,t}$ associated to each unit k. Then :

$$k \in S_t \Leftrightarrow g_{k,t}(\omega_k) \in [0,\pi_{k,t}[$$

We define : $A_{k,t} = g_{k,t}^{-1}[0,\pi_{k,t}[$

$\rightarrow$ indicator function : $I_{k,t}(\omega_k) = \mathbb{1}_{A_{k,t}}(\omega_k)$

Application to Poisson sampling

Sampling

For a separate sample t, for the selection of the sample S_t :

criterion $C_{k,t}(\omega_k)$ = cumulative burden $\Gamma_{k,t-1}(\omega_k)$, and then deduce $g_{k,t}$

For updating a panel : S_u = sample corresponding to the latest update ($u \leq t-1$). We denote $A_{k,u} = g_{k,u}^{-1}[0, \pi_{k,u}[$

First-stage criterion in the calculation of the coordination function :
any decreasing function of the indicator function of $A_{k,u}$.
For example : $C_{k,t}(\omega_k) = 1$ if $\omega_k \in A_{k,u}$ (i.e. $k \in S_u$)
$= 2$ if $\omega_k \notin A_{k,u}$ (i.e. $k \notin S_u$)

To take into account past burdens :
cumulative burden $\Gamma_{k,t-1}$ = secondary criterion
This leads to a certain coordination function $g_{k,t}$.

Application to SSRS

1. Calculation of the approximate $I'_{k,t}(\omega_k)$, by its conditional expectation

$$I'_{k,t}(\omega_k) = E\left(I_{k,t}(\Omega) \middle| \Omega = \Omega_k\right) = b_{k,t}\left(g_{k,t}(\omega_k)\right)$$

where $\Omega = (\Omega_1 \ldots \Omega_k \ldots \Omega_N)$ is a random vector from which we have a realization ω.

2. Approximation by step functions

$I'_{k,t}$ and the cumulative burden function are no longer step functions. These functions are approximated by step functions, which are constant over predefined intervals, obtained by dividing [0, 1[in L equal subintervals.

Then, the procedures are similar to those used in Poisson sampling.

Simulation

Fixed population of size 100.

Initial burden = 0 for each unit.

Selection of 20 samples (simple random sampling).

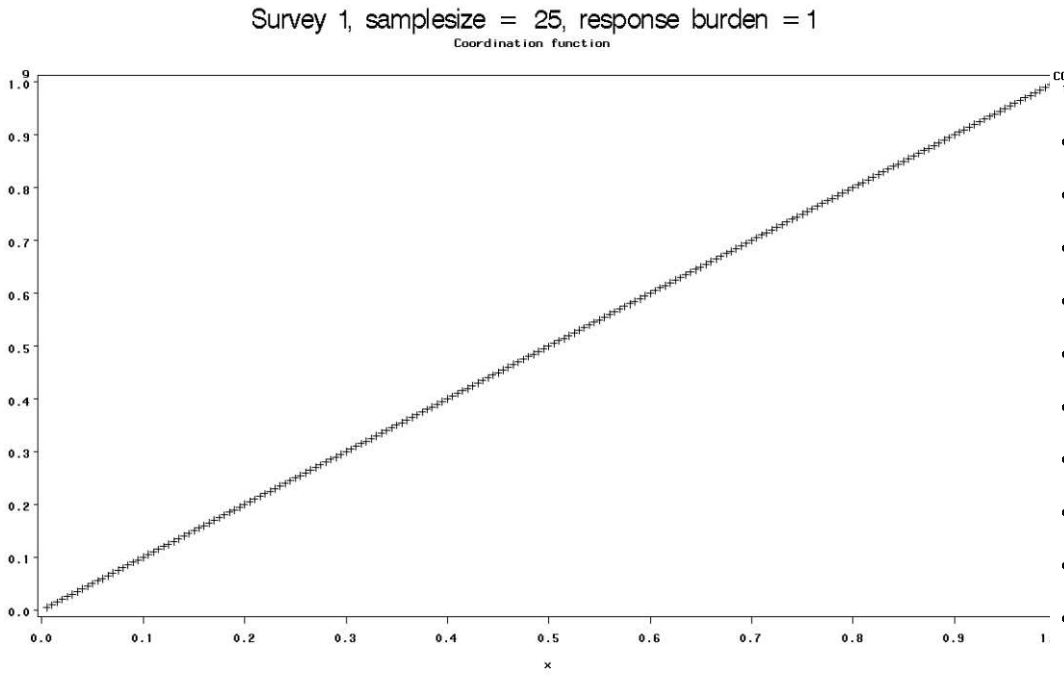
For each sample : sample size = 25, response burden =1,

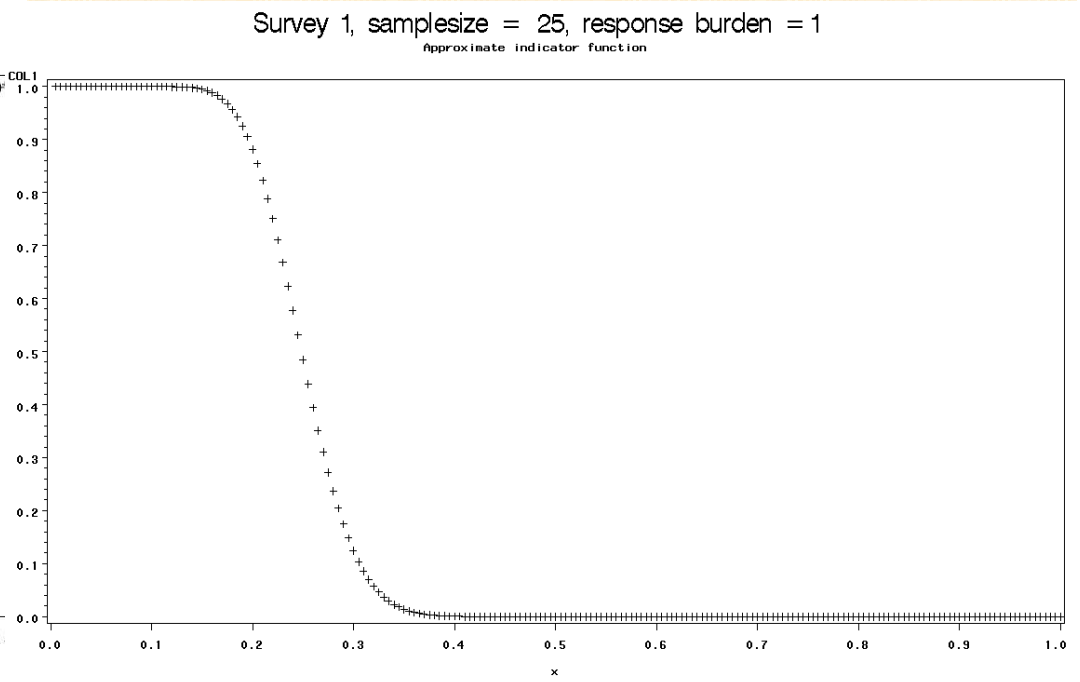
except :

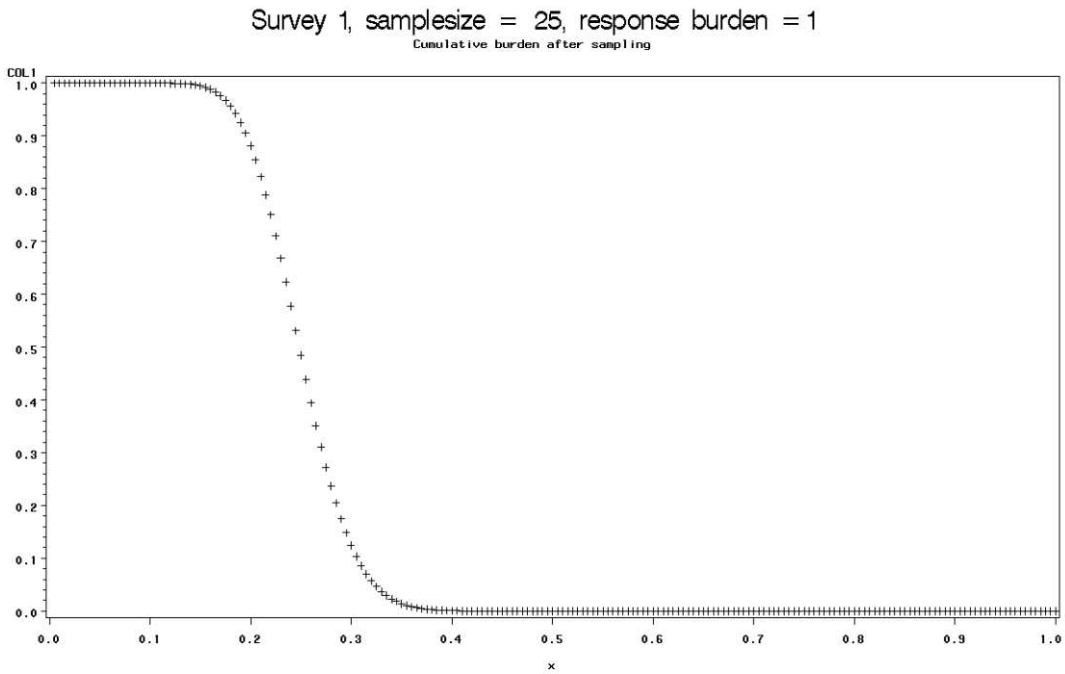
Samples n°3 and n°15 : sample size = 50, response burden = 3

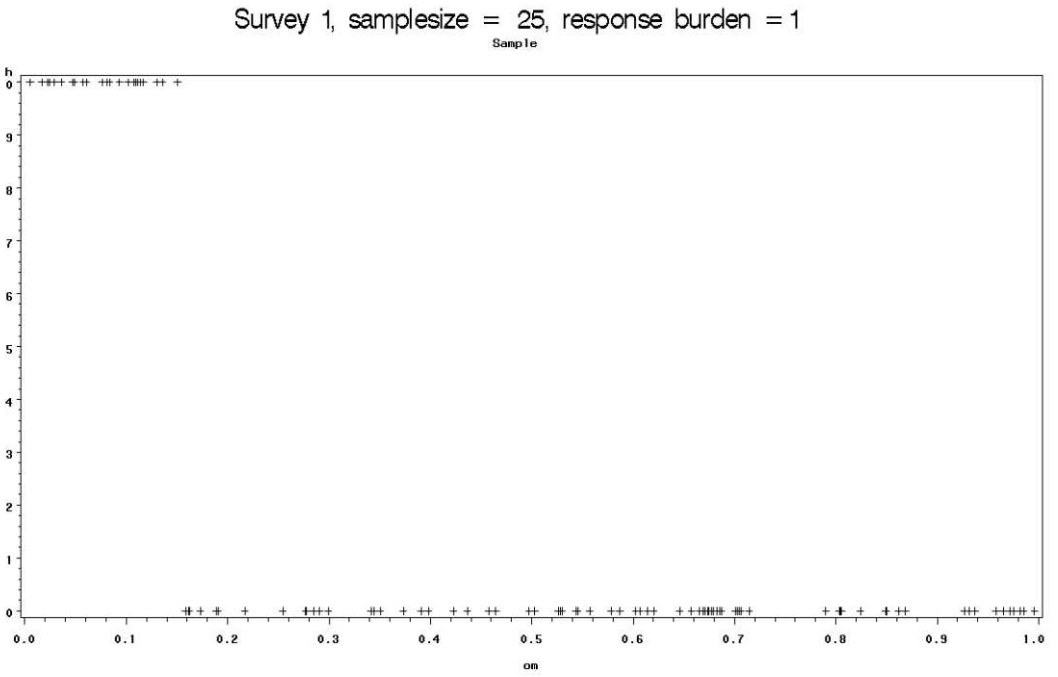
Samples n°10 and n°11 : sample size =10, response burden = 2

→mean cumulative burden = 7.4


Survey 1, samplesize = 25, response burden = 1
Coordination function
x


Survey 1, samplesize = 25, response burden = 1
Approximate indicator function
COL1
x


Survey 1, samplesize = 25, response burden = 1
Cumulative burden after sampling
COL1
x


Survey 1, samplesize = 25, response burden = 1
Sample
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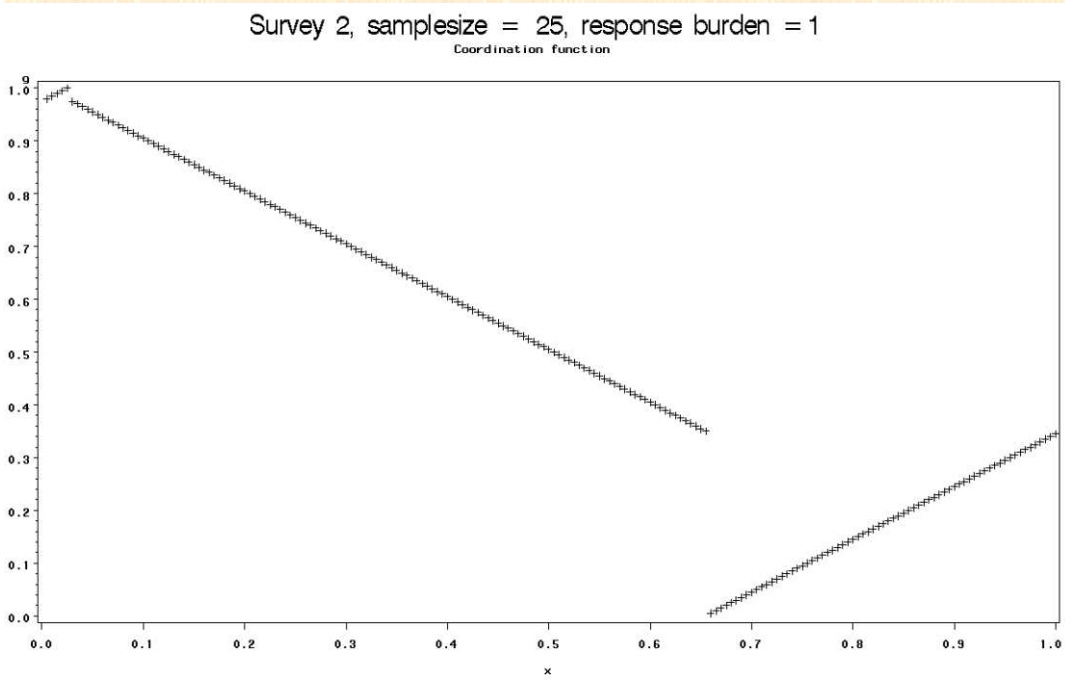
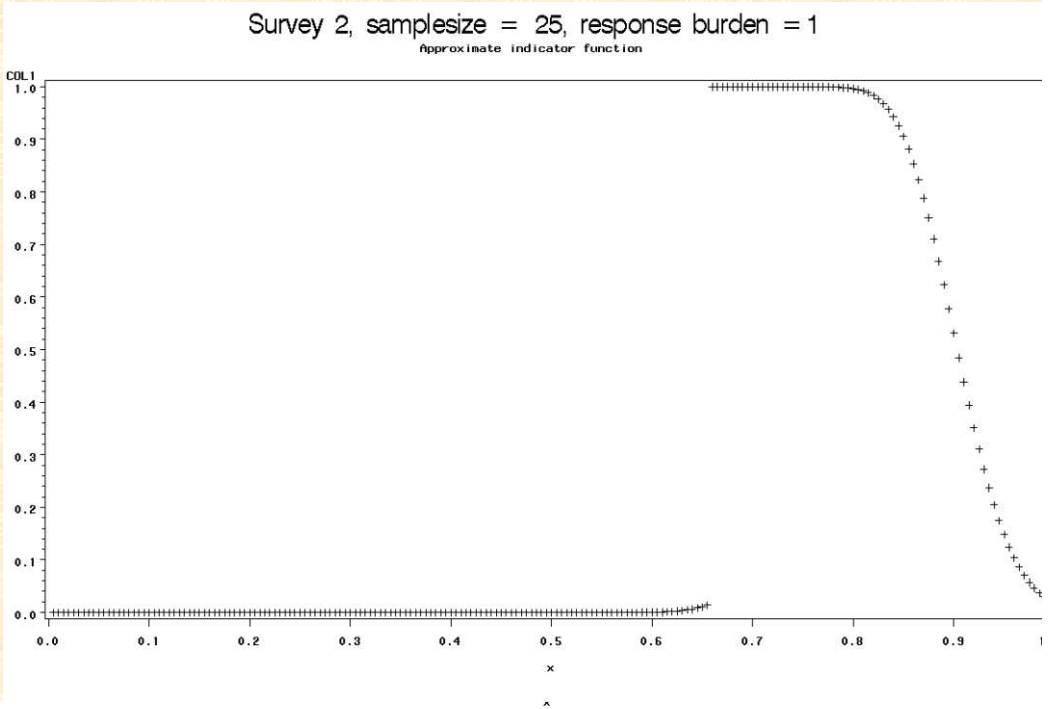
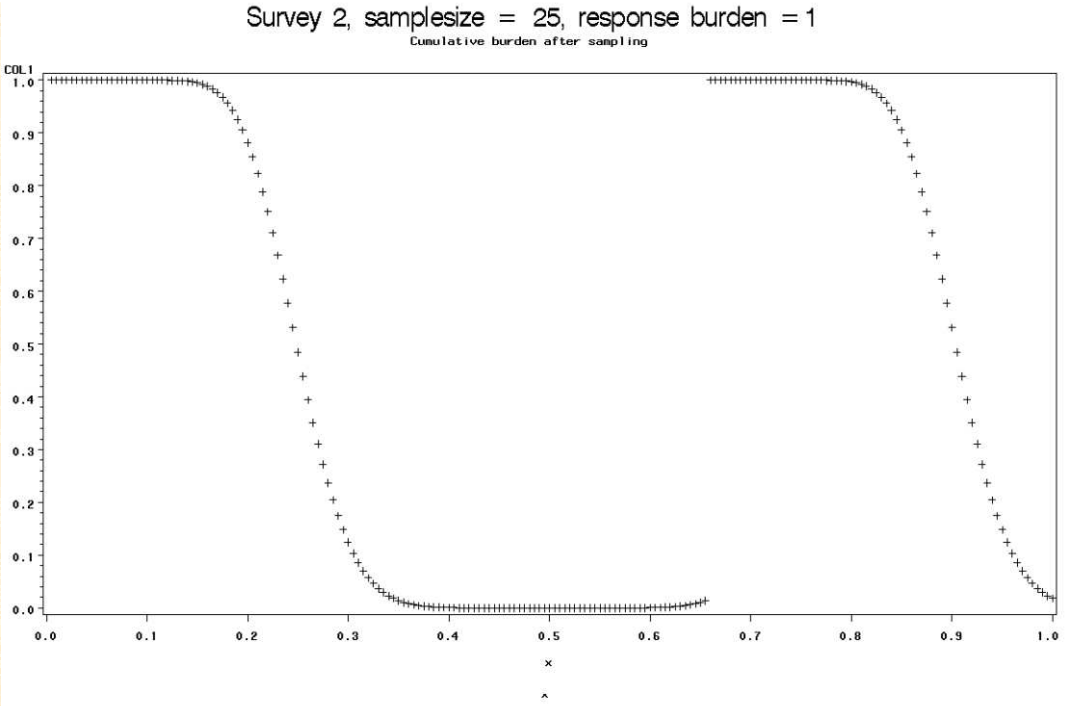
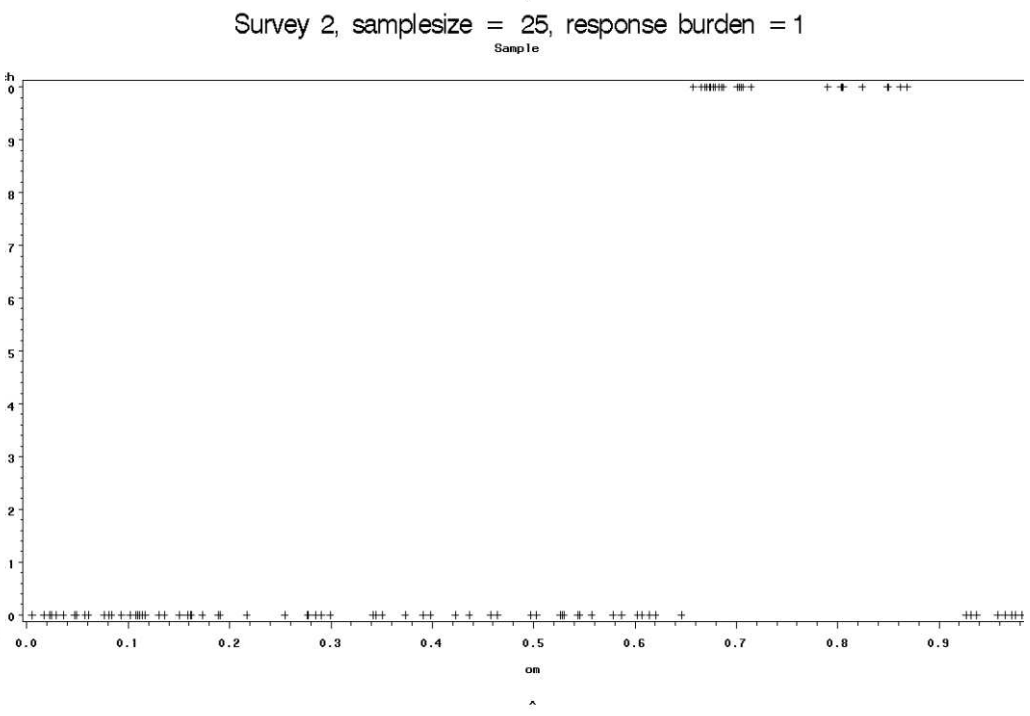
om


Survey 2, samplesize = 25, response burden = 1
Coordination function
x

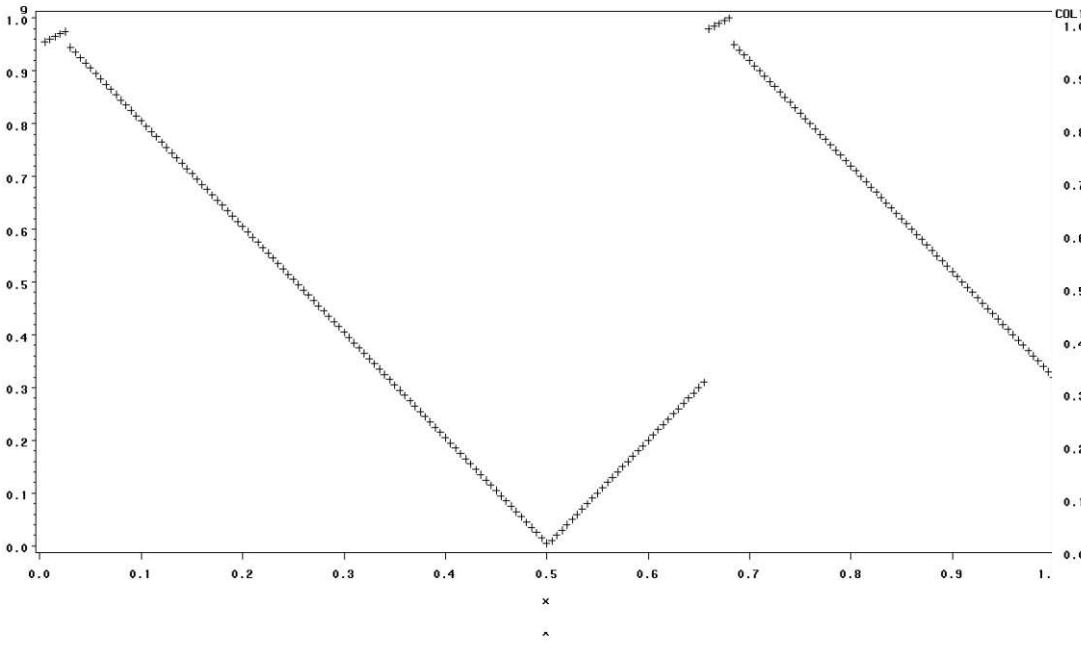

Survey 2, samplesize = 25, response burden = 1
Approximate indicator function
COL1
x


Survey 2, samplesize = 25, response burden = 1
Cumulative burden after sampling
COL1
x


Survey 2, samplesize = 25, response burden = 1
Sample
om

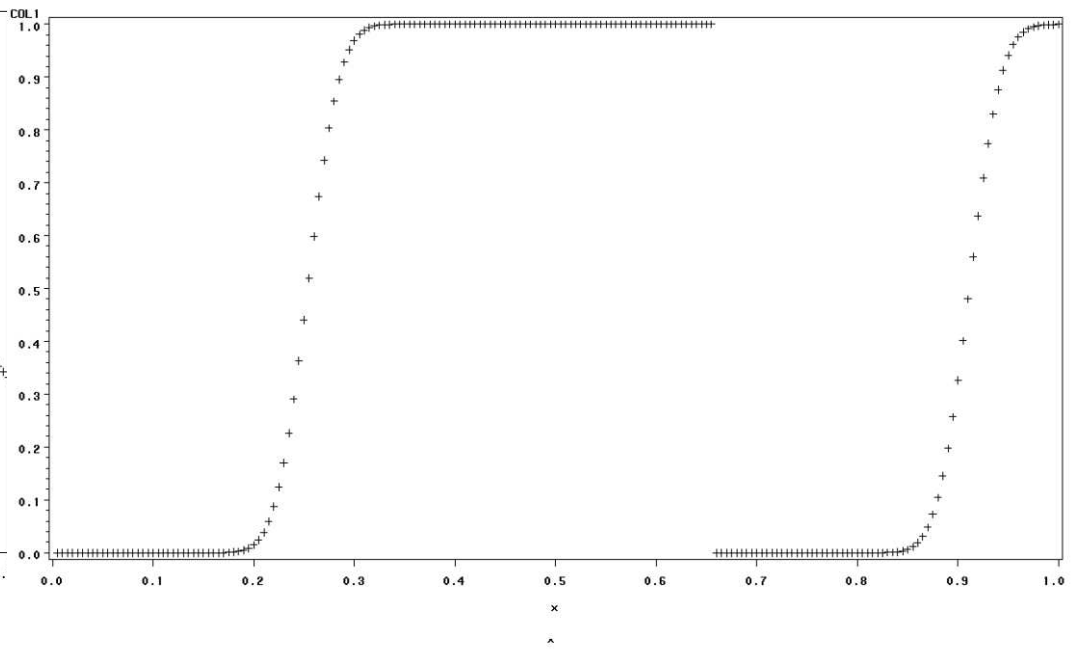
Survey 3, samplesize = 50, response burden = 3

Coordination function



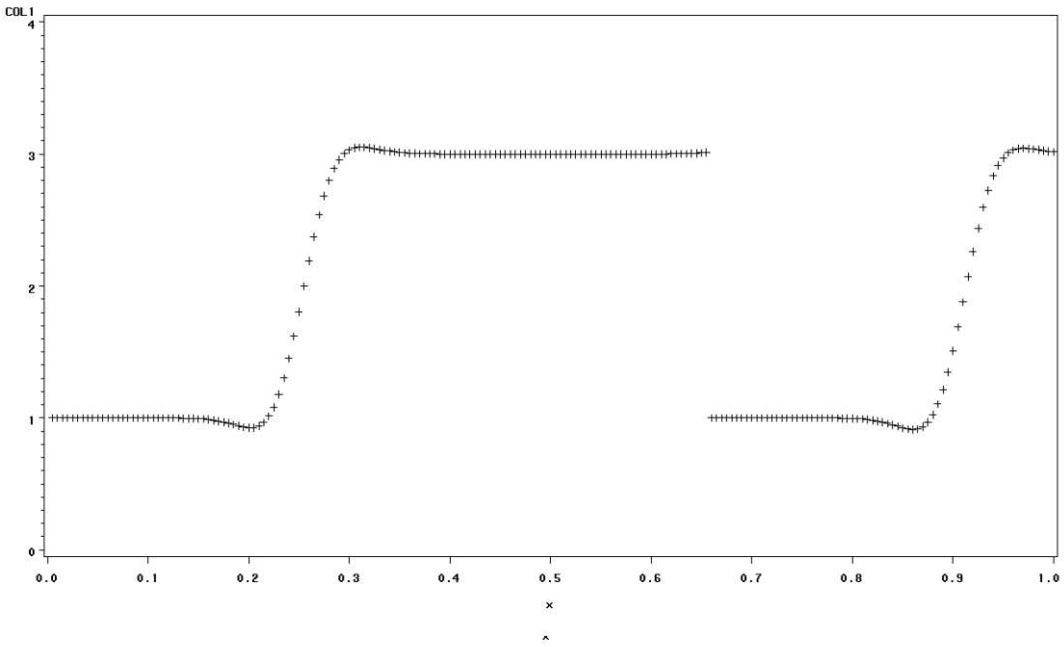
Survey 3, samplesize = 50, response burden = 3

Approximate indicator function



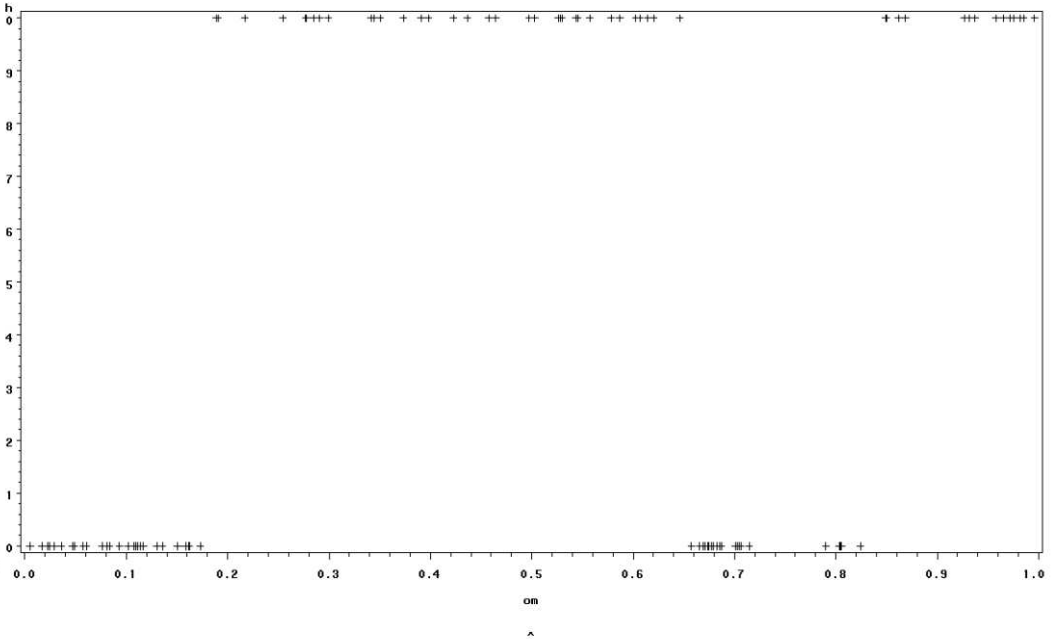
Survey 3, samplesize = 50, response burden = 3

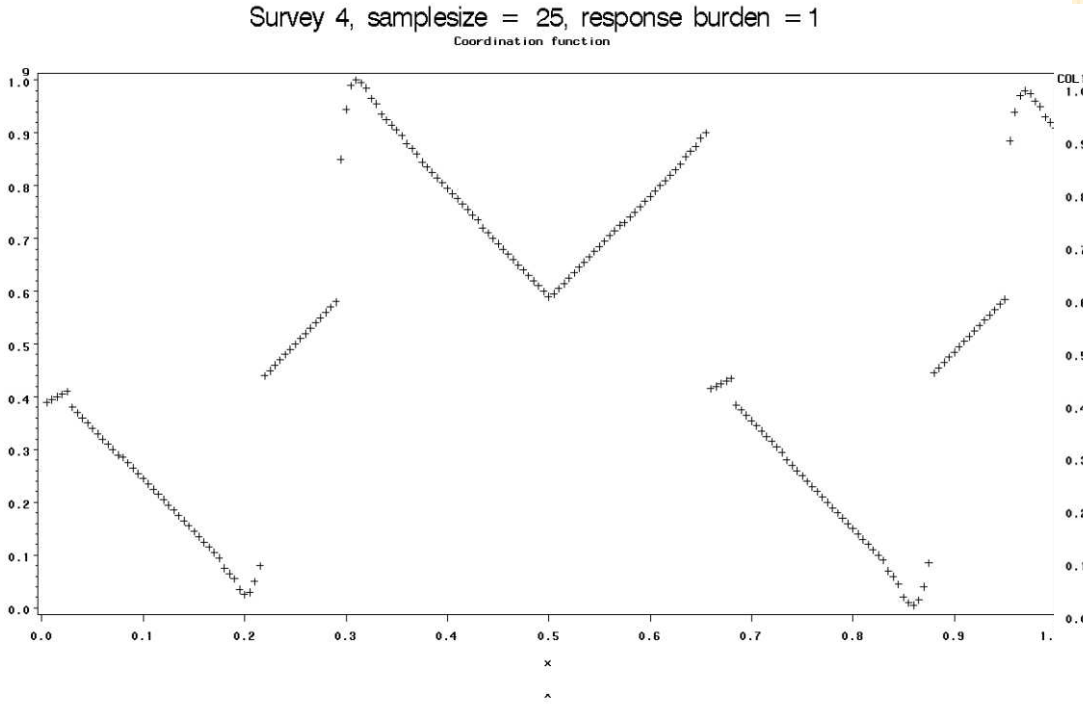
Cumulative burden after sampling

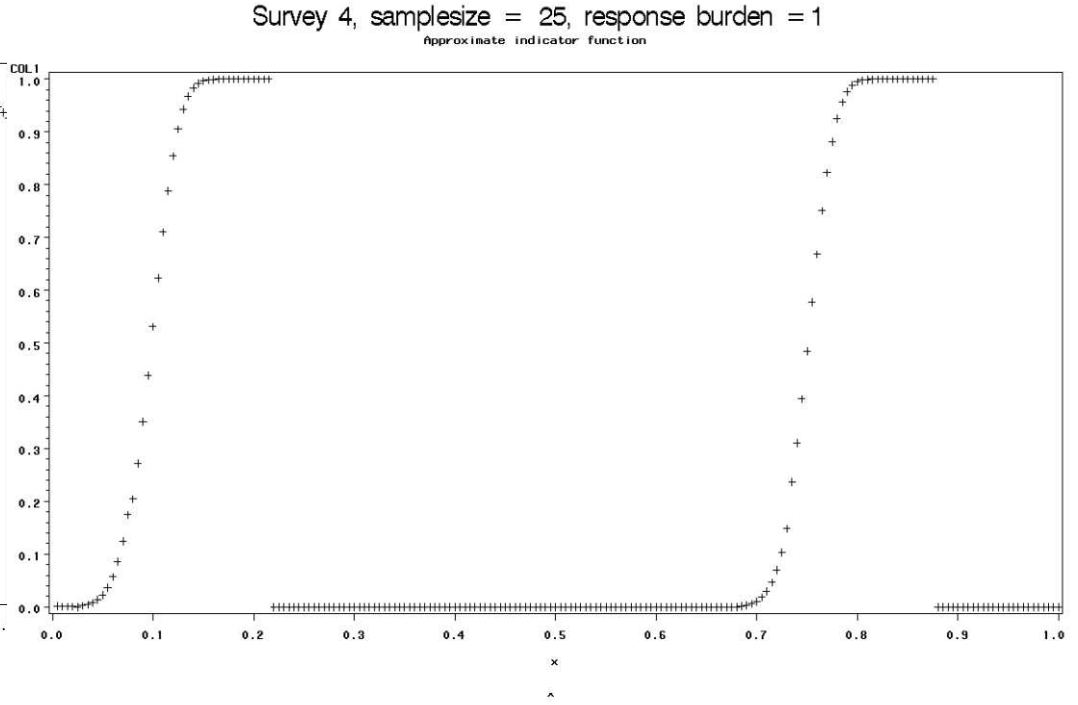


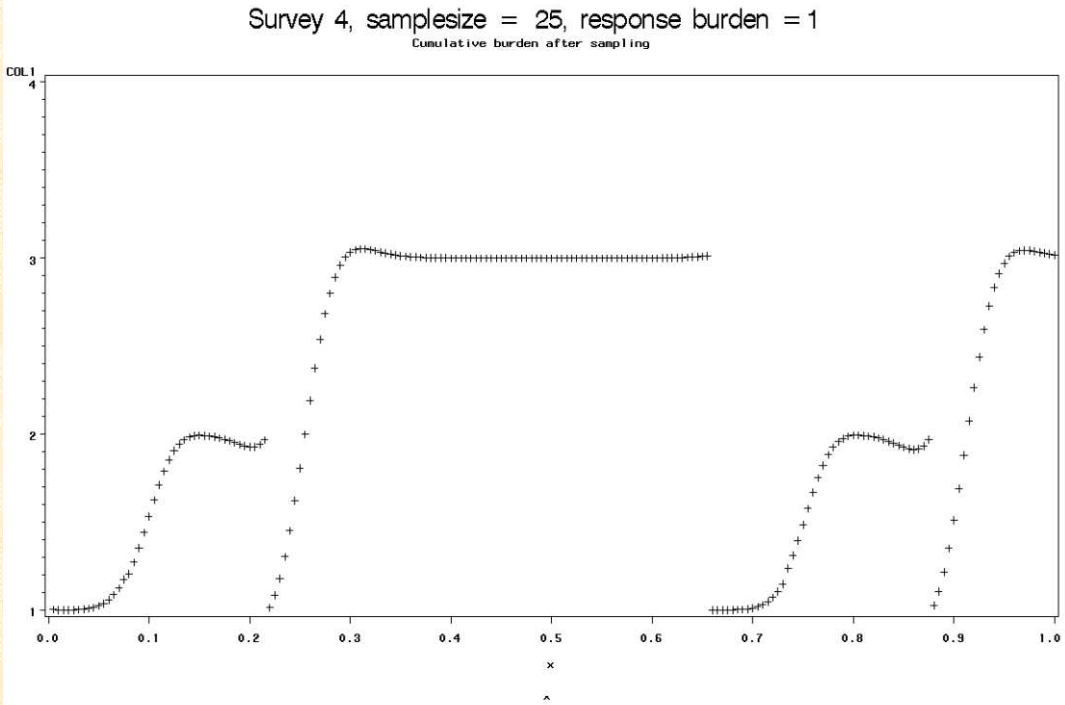
Survey 3, samplesize = 50, response burden = 3

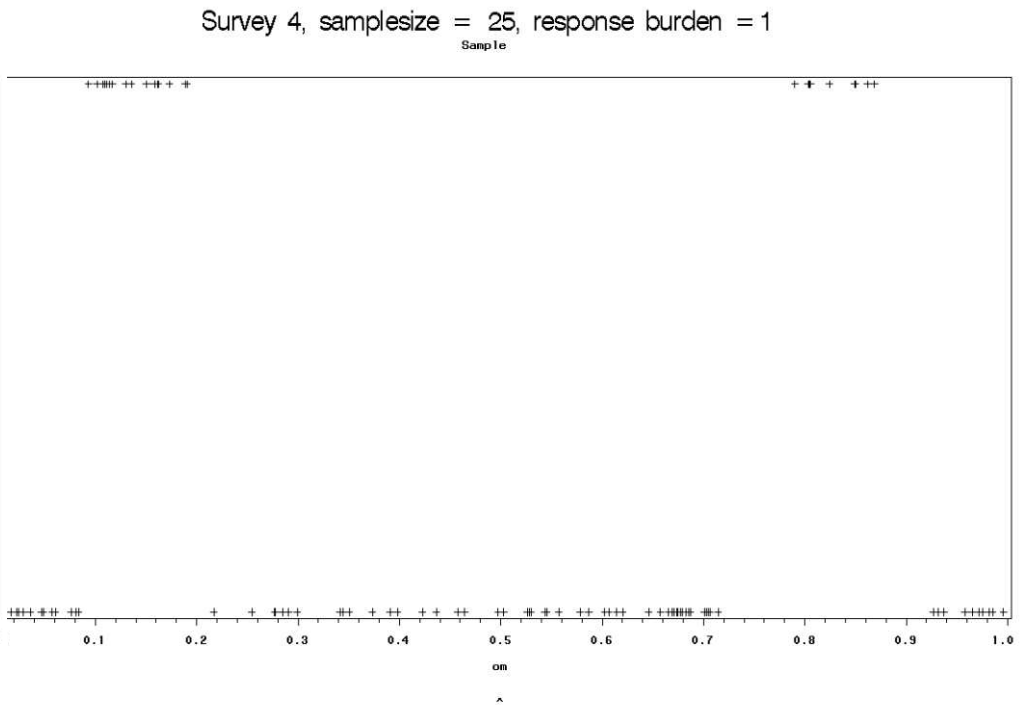
Sample

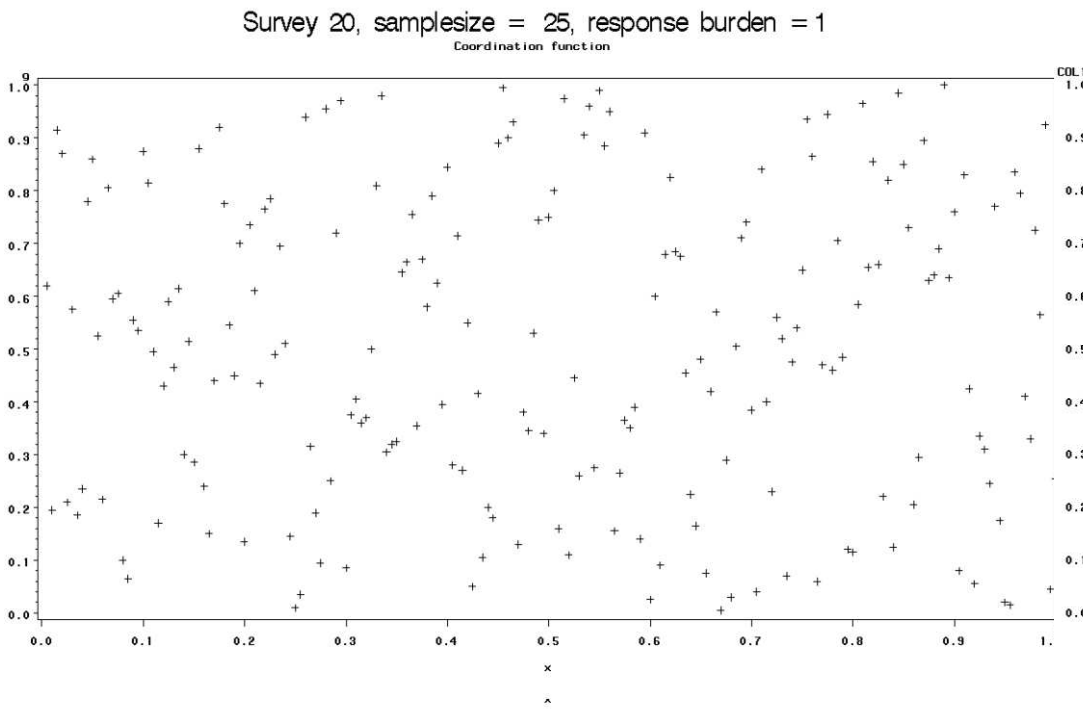


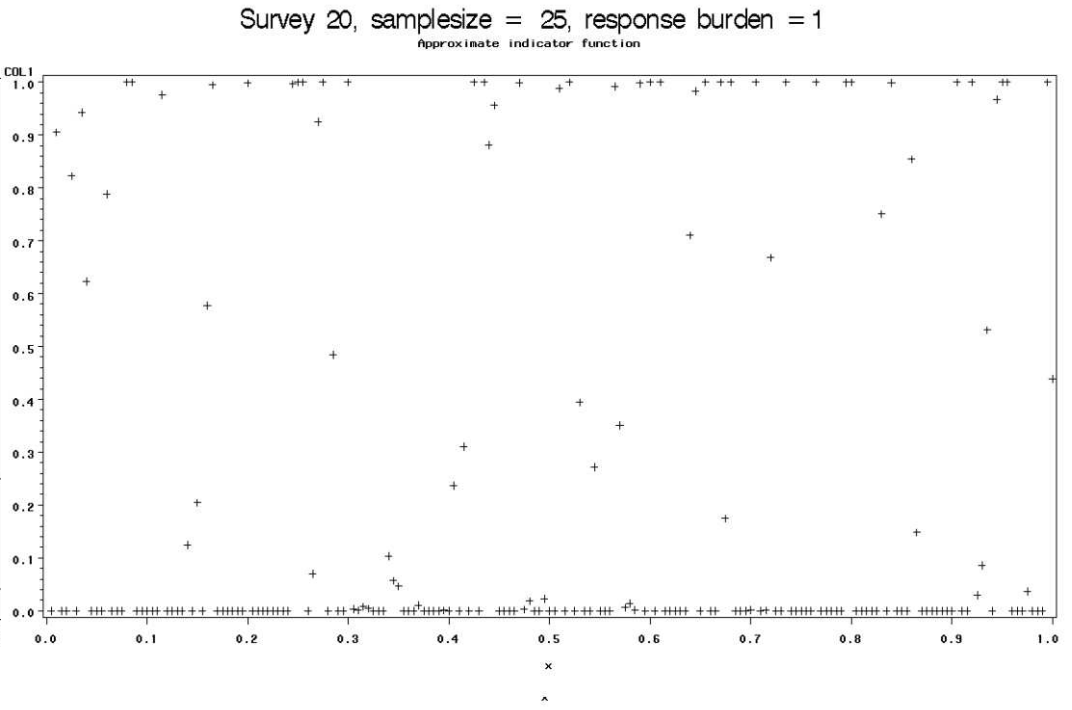

Survey 4, samplesize = 25, response burden = 1
Coordination function
x

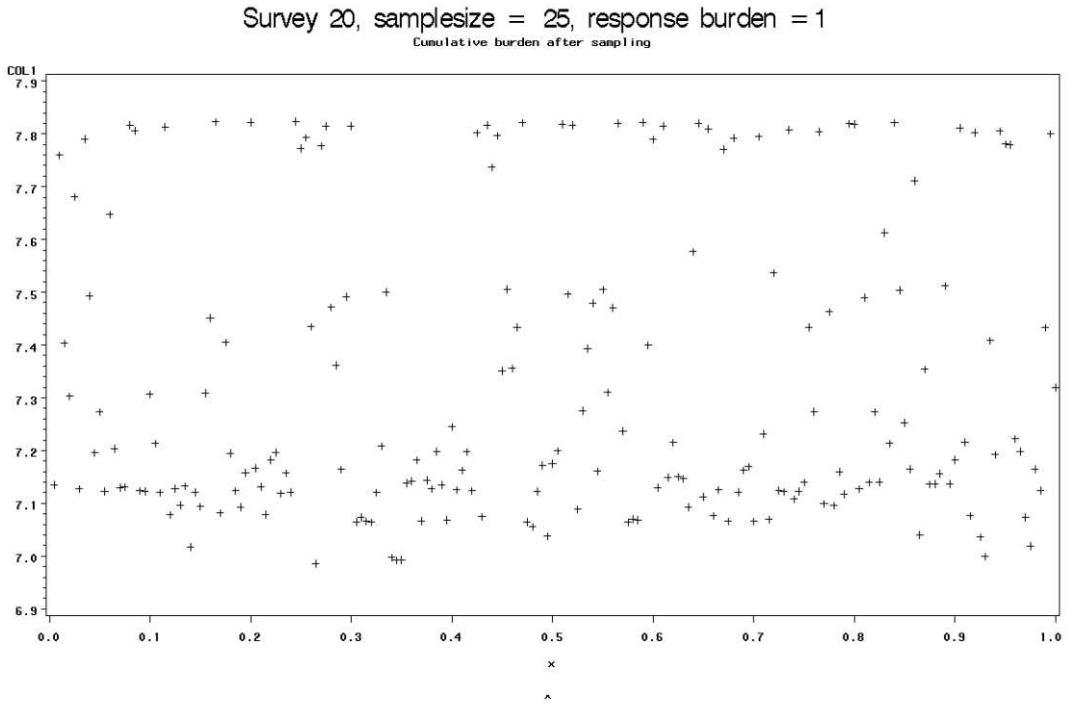

Survey 4, samplesize = 25, response burden = 1
Approximate indicator function
COL1
x

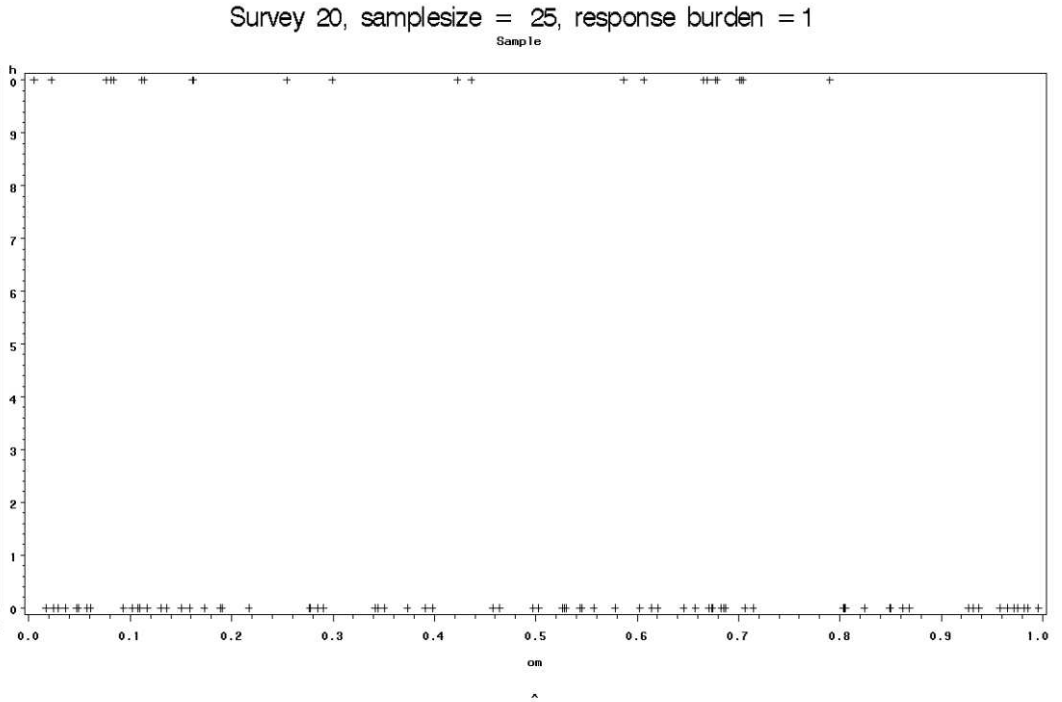

Survey 4, samplesize = 25, response burden = 1
Cumulative burden after sampling
COL1
x


Survey 4, samplesize = 25, response burden = 1
Sample
om


Survey 20, samplesize = 25, response burden = 1
Coordination function
x


Survey 20, samplesize = 25, response burden = 1
Approximate indicator function
COL1
x


Survey 20, samplesize = 25, response burden = 1
Cumulative burden after sampling
COL1
x


Survey 20, samplesize = 25, response burden = 1
Sample
h
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