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Small Area Estimation for Business Surveys at Statistics Canada

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Outline

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Introduction

- Motivation for Small Area Estimation
  - Data users want more data, more details and want them now
  - Auditor General of Canada

    Statistics Canada should assess the feasibility of more fully addressing user needs for data from small areas and subpopulations
- Current environment has led to a need to find new ways
- A possible solution - Small Area Estimation (SAE)

Introduction

- SAE is not new at Statistics Canada
  - Used in for Census Undercoverage since 1991
- Recognizing the potential of SAE, Statistics Canada has developed an SAE prototype
- Business surveys have different characteristics
  - Interest doesn’t necessarily lie in geographical small areas
  - Interest typically lies in detailed industry domains (perhaps crossed with geography)
  - High quality auxiliary data is usually available
- This talk will present the prototype and illustrate its use with the survey of Research and Development in Canadian Industry (RDCI)

SAE Prototype

- Developed primarily by Mike Hidiroglou and Victor Estevao
- Consists of a series of SAS macros and IML modules
- Runs under SAS 9.2 or 9.3 in a Windows environment
- Is available free of charge but with limited support
- Handles both area level and unit level small area models

Area Level Models

- Area level models relate small area means or totals to area specific auxiliary data through models such as

$$\bar{y}_i = \mathbf{z}_i^T \boldsymbol{\beta} + b_i v_i + e_i \qquad\qquad i = 1, \ldots, m$$

  where $\mathbf{z}_i$ is a vector of small area auxiliary data, $\boldsymbol{\beta}$ is a vector of regression parameters, v_i are the small area random effects and e_i are the sampling errors
- Certain assumptions on the model are made (Rao 2003)
- The prototype offers two methods of estimating the model parameters
  - Empirical Best Linear Unbiased Prediction (EBLUP)
  - Hierarchical Bayes (HB)

Area Level - EBLUP Estimation

- The small area model is a general linear mixed model
  - Can appeal to results from classical statistics to define the Best Linear Unbiased Predictor (BLUP) - assumes σ_v^2 known
- EBLUP is obtained by replacing σ_v^2 with an estimate $\hat{\sigma}_v^2$
- Four methods of estimating σ_v^2 are offered
  - Fay-Herriot method - a method of moments estimator
  - Restricted Maximum Likelihood (REML) - requires normality of v_i's
  - Wang-Fuller
  - Adjusted Maximum Likelihood

Area Level - HB Estimation

- The prototype has implemented the HB method using the MCMC method called Gibbs Sampling
- HB methods assume the Normality of the e_i's and v_i's in the area level model
- Under Normality and with σ_v^2 is known, the required conditional distributions can be derived
- Gibbs sampling is used to generate observations from these conditional distributions
- The SA estimates are calculated based on these observations
- For more details, see Rao (2003)

Area Level - HB Estimation

- The simple model assumes $\theta_i = \mathbf{z}_i^T \boldsymbol{\beta} + b_i v_i$, which may not appropriate, so the prototype offers two additional linking models
  - Unmatched log-linear model $log(\theta_i) = \mathbf{z}_i^T \boldsymbol{\beta} + b_i v_i$
  - Unmatched log census undercount model $log\left(\frac{\theta_i}{\theta_i + c_i}\right) = \mathbf{z}_i^T \boldsymbol{\beta} + b_i v_i$ where c_i is the number of units in small area i
- For more details on these conditional distributions, see Estevao et al. (2014)

Unit Level Models

- Unit level models relate a business' values to business specific auxiliary data
  - Auxiliary data must be available at the unit level for all units in the population
- For unit level models, the prototype uses a nested error model

$$y_{ij} = \mathbf{x}_{ij}^T \boldsymbol{\beta} + v_i + e_{ij} \qquad i = 1, \ldots, m, \ j = 1, \ldots, N_i$$

- This model is a special case of the general linear mixed model with block diagonal covariance structure
  - EBLUP methods for area level model are applicable
- Pseudo-EBLUP (uses survey weights) are also available in prototype
  - For details, see Rao (2003)

Additional Output

- Estimates of the MSE of small area estimates
  - Prasad-Rao approach for EBLUP
  - Gibbs sampler for HB
- Diagnostic plots
  - Residual plots
  - Q-Q plots
  - Influential measures

Illustration - Research and Development in Canadian Industry (RDCI)

- Collects information on R&D and personnel engaged in R&D
- Annual survey with $n \approx 2,100$
- Stratified Bernoulli design
- First level of stratification is 55 NAICS group
- Units further stratified by previous R&D expenses
- Sample design is optimized to produce estimates for the 55 groups
- System of National Accounts (SNA) requires estimates for 212 detailed domains

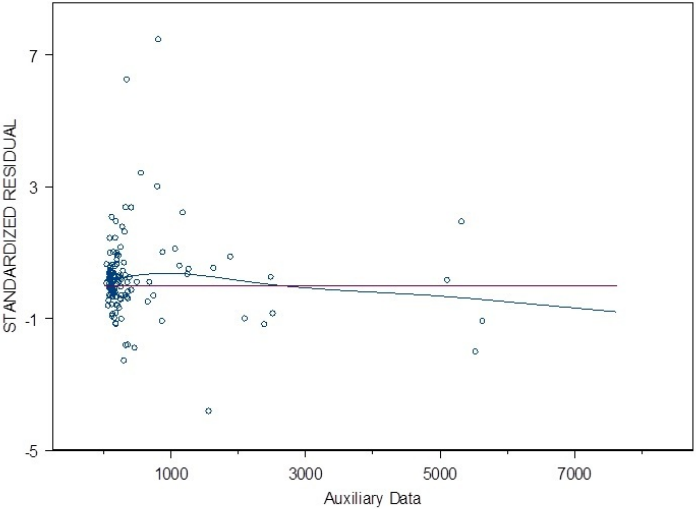
Illustration - RDCI

- Sample of 2,100 can not support this
- Auxiliary data are available from taxation agency
    - Tax credit for Scientific Research and Experimental Development Expenditures
    - Current Intramural Expenses (CIE) available on both survey and auxiliary data source
- SAE prototype used to produce estimates for 212 SNA domains
- The sample covered only 125 of these 212 domains

Illustration - RDCI

- Given availability of unit specific auxiliary data, unit level model was initially tried
  - Outliers severely affected the fit of the model
- Area level model was then used
- EBLUP estimation was used with variance components being estimated via ADM
- Diagnostic plots did not show any serious issues with model

Diagnostic Plots



Diagnostic Plots

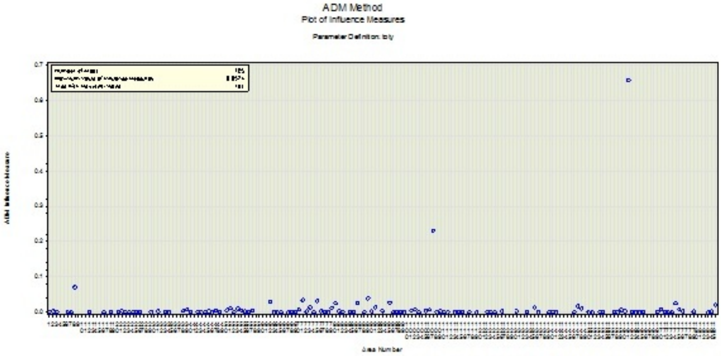
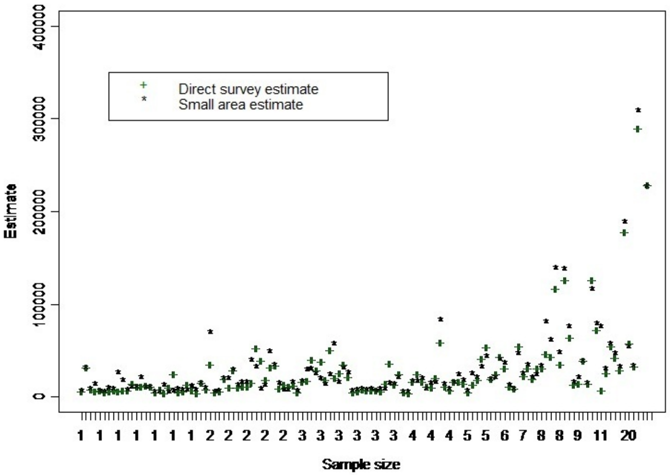


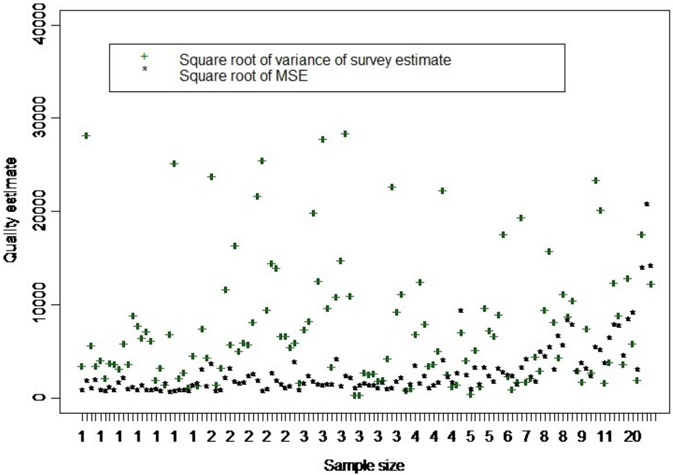
Illustration - RDCI

- SAE was able to produce estimates for 188 of the 212 domains
- In general, the direct survey and SAE estimates were similar
- Turning to MSE/Variance comparisons, the SAE performs much better when the sample sizes are small
- As sample sizes increase the two estimators behave similiarly

Direct vs Small Area Estimates



CV vs MSE


+ Square root of variance of survey estimate
* Square root of MSE
Quality estimate
0
10000
20000
30000
40000
Sample size
1 1 1 1 1 1 1 2 2 2 2 2 3 3 3 3 3 3 4 4 4 5 5 6 7 8 8 9 11 20

Summary

- Based on the RDCI example, SAE seems to be a viable option for business surveys
- Statistics Canada's prototype is a flexible SAE tool which offers users several options
- Statistics Canada is not yet prepared to publish SAE estimates but RDCI will produce SAE estimates for internal use for RY2014